

Yukawa coupling beta-functions in the Standard Model at three loops

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Abstract

We present the results for three-loop beta-functions for Yukawa couplings of heavy Standard Model fermions calculated within the unbroken phase of the model. The calculation is carried out with the help of the MINCER program in a general linear gauge, and the final result is independent of the gauge-fixing parameters. In order to calculate three-point functions, we made use of infrared rearrangement (IRR) trick. Due to the chiral structure of the SM a careful treatment of loops with fermions is required to perform the calculation. It turns out that gauge anomaly cancellation in the SM allows us to obtain the result by means of the semi-naïve treatment of γ_5 .

Keywords: Standard Model, Renormalization Group

The Yukawa couplings being the fundamental parameters of the Standard Model (SM) Lagrangian describe the interactions of quarks and leptons with the Higgs field. Having in mind the discovery of the Higgs boson candidate [1, 2] one can hope that some day the values of Yukawa couplings will be deduced directly from the experimental data (see, e.g., the discussion presented in Ref. [3]). In order to obtain a very precise SM prediction for the running Yukawa couplings at some high energy scale, one usually uses known masses of quarks and leptons [4] since it is the Yukawa interactions that give the fundamental fermions their masses after spontaneous electroweak symmetry breaking. Due to the observed hierarchy of the SM fermion masses the corresponding values are usually defined at different scales, so one inevitably makes use of renormalization group equations (RGE) to connect these scales. The so-called threshold (matching) corrections (see Refs. [5], [6] for the case of running masses and Yukawa couplings) are also very important for extractions of the running SM parameters defined in the minimal subtraction ($\overline{\text{MS}}$) scheme, in which counter-terms and beta-functions have a very simple polynomial structure. It also should be mentioned that contrary to leptons, quarks are not observed as free particles, so the pole mass which is usually associated with the physical mass of a particle, although being a gauge-invariant and infrared finite quantity [7, 8], suffers from the so-called renormalon ambiguity [9, 10]. This intrinsic uncertainty of the quark pole mass is estimated to be of the order of $\Lambda_{QCD} \simeq 200$ MeV. For the top-quark one usually neglects this uncertainty since the corresponding mass $M_t = 175$ GeV is much bigger than Λ_{QCD} . Moreover, it is generally believed that due to the short lifetime the t -quark does not have time to hadronize, so the notion of the pole mass can be used in this case. According to the recent studies of the relation between the $\overline{\text{MS}}$ -running mass of the top quark and the corresponding pole mass, electroweak corrections can become important and for the observed value of the Higgs boson mass can severely cancel the QCD contribution [6]. As a consequence, theoretical uncertainty in determination of the top-quark Yukawa coupling is reduced, thus calling for more precise determination of the corresponding RGEs. For all the other quarks one usually uses the $\overline{\text{MS}}$ masses defined initially in the context of QCD (see, e.g., Ref. [11] and references therein).

The SM Higgs boson with $M_h = 125$ GeV decays predominately into the $b\bar{b}$ pairs. In spite of the fact that this decay mode is very hard to observe due to the enormous QCD background it is obvious that the

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precise value of the corresponding Yukawa coupling is required to test whether the SM correctly describes Nature at the electroweak scale. If one considers leptonic decays of the Higgs boson, the most promising decay channel is the tree-level decay to a tau-anti-tau pair. These facts somehow motivate our study of the three-loop contribution to the corresponding Yukawa beta-functions.

Moreover, we would like to stress here that a separate study of the high-energy behavior of the SM can also be of great importance. One can use RGE to find the scale where New Physics should enter the game, e.g., to unify the interactions or stabilize the Higgs potential [12–17].

One- and two-loop results for SM beta functions have been known for quite a long time [18–31] and are summarized in [32].

Not long ago full three-loop gauge coupling beta-functions were calculated [33, 34]. The beta-functions for the Higgs self-coupling and top Yukawa coupling were also considered at three loops [35]. However, in Ref. [35], all the electroweak couplings were neglected together with the Yukawa couplings of other SM fermions. In this paper, we provide the full analytical result for the three-loop beta-functions of the strongest Yukawa couplings y_t, y_b, y_τ for the three heaviest SM fermions (top, bottom, and tau). We take into account all the interactions of the SM.

Let us briefly define our notation. The full Lagrangian of the unbroken SM which was used in this calculation is given in our previous paper [34]. However, we do not keep the full flavor structure of Yukawa interactions but use the following simple Lagrangian which describes fermion-higgs interactions

$$\mathcal{L}_{\text{Yukawa}} = -y_t(\bar{Q}\Phi^c)t_R - y_b(\bar{Q}\Phi)b_R - y_\tau(\bar{L}\Phi)\tau_R + \text{h.c.} \quad (1)$$

with $Q = (t, b)_L$, and $L = (\nu_\tau, \tau)_L$ being SU(2) doublets of left-handed fermions of the third generation, u_R , t_R , and τ_R are the corresponding right-handed counter parts¹. The Higgs doublet Φ with $Y_W = 1$ has the following decomposition in terms of the component fields:

$$\Phi = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(h + i\chi) \end{pmatrix}, \quad \Phi^c = i\sigma^2\Phi^\dagger = \begin{pmatrix} \frac{1}{\sqrt{2}}(h - i\chi) \\ -\phi^- \end{pmatrix}. \quad (2)$$

Here a charge-conjugated Higgs doublet is introduced Φ^c with $Y_W = -1$.

For loop calculations it is convenient to define the following quantities:

$$a_i = \left(\frac{5}{3} \frac{g_1^2}{16\pi^2}, \frac{g_2^2}{16\pi^2}, \frac{g_s^2}{16\pi^2}, \frac{y_t^2}{16\pi^2}, \frac{y_b^2}{16\pi^2}, \frac{y_\tau^2}{16\pi^2}, \frac{\lambda}{16\pi^2}, \xi_G, \xi_W, \xi_G \right), \quad (3)$$

where we use the SU(5) normalization of the U(1) gauge coupling g_1 . We also stress that the calculation is carried out in a general linear R_ξ gauge, so the renormalization of all three gauge-fixing parameters ξ_G , ξ_W , and ξ_B , given in Ref. [34], is required.

The Yukawa beta-functions are extracted from the corresponding renormalization constants which relate bare couplings to the renormalized ones in the $\overline{\text{MS}}$ -scheme. The latter are found with the help of the following formulae:

$$Z_{y_f} = \frac{Z_{ffh}}{\sqrt{Z_{f_L}Z_{f_R}Z_h}} \quad \text{or} \quad Z_{y_f} = \frac{Z_{ff\chi}}{\sqrt{Z_{f_L}Z_{f_R}Z_\chi}}, \quad f = t, b, \tau, \quad (4)$$

where Z_{ffh} and $Z_{ff\chi}$ are the renormalization constants for the three-point vertices involving two fermions f and the higgs h or the would-be goldstone boson χ , respectively. The renormalization constants Z_{f_L} , Z_{f_R} for left- and right-handed fermions, and Z_χ and Z_h for the neutral components of the Higgs doublet are obtained from the corresponding self-energy diagrams.

In order to extract a three-loop contribution to the considered renormalization constants, it is sufficient to know the two-loop results for the gauge and Yukawa couplings and the one-loop expression for the Higgs self-interaction.

¹Here we assume that neutrinos are massless.

The relation between the bare and renormalized parameters can be written in the following way

$$a_{k,\text{Bare}}\mu^{-2\rho_k\epsilon} = Z_{a_k}a_k(\mu) = a_k + \sum_{n=1}^{\infty} c_k^{(n)} \frac{1}{\epsilon^n}, \quad (5)$$

where $\rho_k = 1$ for the gauge and Yukawa constants, $\rho_k = 2$ for the scalar quartic coupling constant, and $\rho_k = 0$ for the gauge fixing parameters. The bare couplings are defined within the dimensionally regularized [36] theory with $D = 4 - 2\epsilon$. The four-dimensional beta-functions, denoted by β_i , are defined via²

$$\beta_i(a_k) = \left. \frac{da_i(\mu, \epsilon)}{d \ln \mu^2} \right|_{\epsilon=0}, \quad \beta_i = \beta_i^{(1)} + \beta_i^{(2)} + \beta_i^{(3)} + \dots \quad (6)$$

with $\beta_i^{(l)}$ being the l -loop contribution to the beta-function for a_i . The expression for β_i can be extracted from the corresponding renormalization constants (5) with the help of

$$\beta_i = \sum_l \rho_l a_l \frac{\partial c_i^{(1)}}{\partial a_l} - \rho_i c_i^{(1)}. \quad (7)$$

Here, again, a_i stands for both the gauge couplings and the gauge-fixing.

It should be noted that the divergent part of the considered three-point functions should resemble the tree-level structure

$$\Delta \mathcal{L} = -\frac{y_f}{\sqrt{2}} \bar{f} f h - i \frac{y_f}{\sqrt{2}} \bar{f} \gamma_5 f \chi. \quad (8)$$

Since we separately consider the contributions to the $\bar{f}_L f_R \phi$ and $\bar{f}_R f_L \phi$ vertices ($\phi = h, \chi$), we have to be sure that the corresponding divergencies sum up to give the unit matrix in the case of the Higgs boson or γ_5 in the case of χ . This serves as an additional self-consistency check of our result.

It turns out that due to the gauge symmetry the Higgs field h and the would-be goldstone boson χ renormalize in the same way so that $Z_\chi = Z_h$. Moreover, the same reasoning can be applied to the considered Yukawa vertices giving $Z_{ffh} = Z_{ff\chi}$. This fact was also checked by explicit calculation at the three-loop level.

Actually, it is not trivial to satisfy these two requirements (to conserve the chiral structure of the Lagrangian and do not break the gauge invariance) at three loops. Both the issues are related to the γ_5 problem present in dimensionally regularized theories. It is known from the literature (see, e.g., Ref. [37] and recent explicit calculation [35]) that the traces with an odd number of γ_5 appearing for the first time in the three-loop diagrams require special treatment. We closely follow the semi-naive approach presented in Ref. [38]. First of all, we anticommute γ_5 with other matrices and use $\gamma_5^2 = 1$. In the “even” traces all γ_5 are contracted with each other, so the corresponding traces can be treated naively in dimensional regularization. In “odd” traces we are left with only one γ_5 . These traces are evaluated as in four dimensions and produce totally antisymmetric tensors via the relation

$$\text{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_5) = -4i \epsilon^{\mu\nu\rho\sigma} \quad (9)$$

with $\epsilon^{0123} = 1$. It is obvious that these tensors can give a nontrivial contribution only if they are contracted with each other. In a fully consistent calculation, this contraction should be carried out only after the renormalization, when one can safely take the limit $\epsilon \rightarrow 0$. However, one can try to use the four-dimensional formula

$$\epsilon^{\mu\nu\rho\sigma} \epsilon_{\alpha\beta\gamma\delta} = -\mathcal{T}_{[\alpha\beta\gamma\delta]}^{\mu\nu\rho\sigma}, \quad \mathcal{T}_{\alpha\beta\gamma\delta}^{\mu\nu\rho\sigma} = \delta_\alpha^\mu \delta_\beta^\nu \delta_\gamma^\rho \delta_\delta^\sigma, \quad (10)$$

to contract two ϵ -tensors originating from different fermion traces *before* the renormalization. In (10) the square brackets denote complete antisymmetrization and δ_α^μ is the Kronecker delta. Due to the fact that

²For the Yukawa couplings it is also convenient to consider the running of the coupling itself. It is obvious that $\beta_{y_i} = (\beta_i/a_i) y_i/2$ with $i = t, b, \tau$.

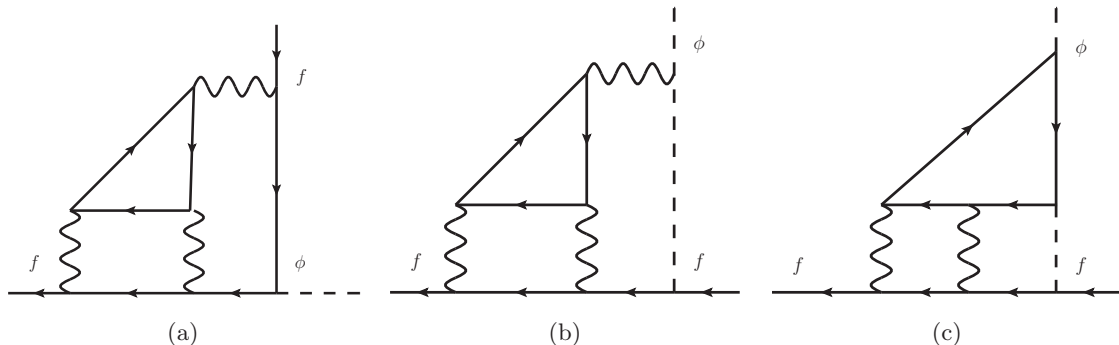


Figure 1: Sample Feynman diagrams giving rise to a non-trivial contribution due to the fermion traces with an odd number of γ_5 . The second trace arises due to the projector applied to the external fermion line. Solid, dashed and wavy lines correspond to fermions, scalar and gauge bosons. In our calculation, we consider vertices with $f = t, b, \tau$ and $\phi = h, \chi$. Due to gauge anomaly cancellation the contributions from the first two diagrams are summed up to give zero if one takes into account all the SM fermions circulating in the triangle loops. As a consequence, we are left with the third topology which produces only single poles in the regularization parameter ϵ .

the metric tensors in the left-hand side of Eq. (10) are treated as D -dimensional ones, in this case the result of contraction can differ from the correct one only via the terms proportional to $D - 4$. If the considered diagrams produce only single poles in ϵ , then the above-mentioned difference contributes only to the finite terms which we can neglect in our calculation³. This kind of approach (semi-naive treatment of γ_5) was used in Refs. [38] and [35]. However, if a diagram gives rise to higher poles in ϵ , then the formal manipulations with antisymmetric tensors affect also lower poles and, in general, lead to a wrong result unless special (finite) counter-terms are introduced (see, e.g., [39]). In order to figure out whether semi-naive treatment is sufficient for our calculation, we separately evaluate contributions arising from the contraction (10). It turns out that some of the three-loop diagrams, both the fermion self-energies and vertices, give rise to the dangerous high-order poles in ϵ (see Figs. 1a and 1b). Nevertheless, taking into account gauge anomaly cancellation in the SM [40, 41] one can check that all these terms sum up to zero. It is worth mentioning that in the SM we have to put the number of colors to be equal to three ($N_c = 3$) to satisfy the anomaly cancellation conditions. In the end, we are left with diagrams presented in Fig. 1c, which produce only single poles and give a nontrivial contribution to the final result (this kind of diagrams was also discussed in Ref. [35]).

As in our previous calculations we use **LanHEP** [42] and **FeynArts** [43] to generate relevant Feynman diagrams and the **FORM** [44] packages **MINCER** [45] and **COLOR** [46] to obtain the resulting expression. Since **MINCER** is aimed only at self-energy type diagrams, we made use of the infrared rearrangement (IRR) trick [47] to evaluate the three-point Green functions by setting the momentum, which enters through the higgs external line, to zero⁴. A special script was written that automatically maps the **FeynArts** three-point vertices with zero external momentum to the **MINCER** topologies.

As a result of our calculation we obtain the expressions⁵ for the three-loop Yukawa coupling beta-functions of the heavy SM fermions. Due to lack of space we present here only the three-loop result for the top quark Yukawa coupling ($\hat{\lambda} \equiv a_\lambda$):

$$\frac{\beta_t^{(1)}}{a_t} = -8a_s + \frac{9a_t}{2} + a_\tau + \frac{3a_b}{2} - \frac{9a_2}{4} - \frac{17a_1}{20}, \quad (11)$$

³This is due to the fact that such a complication arises only in the three-loop diagrams.

⁴A similar method was successfully used in our previous calculations [48, 49].

⁵All the results, including necessary renormalization constants, can be found online as ancillary files of the **arXiv** version of the paper.

$$\begin{aligned}\frac{\beta_t^{(2)}}{a_t} &= a_s^2 \left(\frac{80n_G}{9} - \frac{404}{3} \right) + 36a_s a_t + 6\hat{\lambda}^2 - 12a_t \hat{\lambda} - 12a_t^2 + 4a_s a_b + 9a_2 a_s + \frac{19a_1 a_s}{15} \\ &+ a_2^2 \left(n_G - \frac{35}{4} \right) + a_1^2 \left(\frac{29n_G}{45} + \frac{9}{200} \right) - \frac{9a_\tau a_t}{4} - \frac{11a_b a_t}{4} + \frac{225a_2 a_t}{16} + \frac{393a_1 a_t}{80} \\ &- \frac{9a_\tau^2}{4} + \frac{5a_b a_\tau}{4} + \frac{15a_2 a_\tau}{8} + \frac{15a_1 a_\tau}{8} - \frac{a_b^2}{4} + \frac{99a_2 a_b}{16} + \frac{7a_1 a_b}{80} - \frac{9a_1 a_2}{20},\end{aligned}\quad (12)$$

$$\begin{aligned}\frac{\beta_t^{(3)}}{a_t} &= a_s^3 \left(\frac{1120n_G^2}{81} + \frac{640n_G \zeta_3}{3} + \frac{8864n_G}{27} - 2498 \right) + a_s^2 a_t \left(-54n_G - 228\zeta_3 + \frac{4799}{6} \right) \\ &+ 16a_s a_t \hat{\lambda} - 157a_s a_t^2 + a_t^3 \left(\frac{27\zeta_3}{2} + \frac{339}{8} \right) - 36\hat{\lambda}^3 + \frac{15a_t \hat{\lambda}^2}{4} + 198a_t^2 \hat{\lambda} \\ &+ a_1 a_s^2 \left(-\frac{176n_G \zeta_3}{15} + \frac{88n_G}{9} - \frac{127}{60} \right) + a_2 a_s^2 \left(-48n_G \zeta_3 + 38n_G + \frac{531}{4} \right) \\ &+ a_s^2 a_b \left(-\frac{14n_G}{3} - 44\zeta_3 - \frac{277}{2} \right) + a_1^2 a_s \left(-\frac{748n_G \zeta_3}{75} + \frac{5281n_G}{900} - \frac{1187}{300} \right) \\ &+ a_2^2 a_s \left(-36n_G \zeta_3 + \frac{57n_G}{4} + 66 \right) + a_2 a_s a_t (180\zeta_3 - 168) + a_1 a_s a_t \left(36\zeta_3 - \frac{126}{5} \right) \\ &+ a_1 a_s a_b \left(-\frac{28\zeta_3}{5} - \frac{457}{30} \right) + a_s a_b a_t (27 - 32\zeta_3) + a_s a_b^2 (82 - 64\zeta_3) + \frac{5a_s a_\tau a_t}{2} \\ &+ a_2 a_s a_b \left(-108\zeta_3 - \frac{27}{2} \right) - \frac{43}{6} a_s a_b a_\tau - \frac{321}{20} a_1 a_2 a_s + a_2 a_b^2 \left(\frac{63\zeta_3}{2} - \frac{2283}{32} \right) \\ &+ a_2^3 \left(\frac{50n_G^2}{9} + 45n_G \zeta_3 - \frac{1139n_G}{144} + \frac{45\zeta_3}{8} - \frac{14677}{576} \right) + a_1 a_2 a_t \left(\frac{369\zeta_3}{20} + \frac{8097}{640} \right) \\ &+ a_1 a_2^2 \left(-\frac{9n_G \zeta_3}{5} - \frac{9n_G}{80} - \frac{27\zeta_3}{40} + \frac{927}{320} \right) + a_2 a_b a_\tau \left(9\zeta_3 - \frac{153}{8} \right) + a_2 a_\tau^2 \left(9\zeta_3 - \frac{315}{16} \right) \\ &+ a_1^2 a_2 \left(-\frac{51n_G \zeta_3}{25} + \frac{241n_G}{400} - \frac{153\zeta_3}{200} + \frac{3243}{1600} \right) + a_1 a_\tau a_t \left(\frac{24\zeta_3}{5} - \frac{63}{5} \right) \\ &+ a_1^3 \left(\frac{146n_G^2}{81} - \frac{323n_G \zeta_3}{75} + \frac{53413n_G}{10800} - \frac{153\zeta_3}{1000} + \frac{18103}{24000} \right) + a_b^3 \left(\frac{9\zeta_3}{2} + \frac{477}{16} \right) \\ &+ a_\tau^3 \left(3\zeta_3 + \frac{71}{16} \right) + a_1 a_2 a_b \left(\frac{27\zeta_3}{10} + \frac{747}{128} \right) + a_1 a_b^2 \left(\frac{19\zeta_3}{10} - \frac{959}{160} \right) + a_1 a_b a_t \left(\frac{\zeta_3}{2} - \frac{1383}{160} \right) \\ &+ a_1^2 a_t \left(-\frac{115n_G}{16} - \frac{93\zeta_3}{200} - \frac{44179}{19200} \right) + a_1^2 a_b \left(-\frac{23n_G}{240} - \frac{199\zeta_3}{200} - \frac{35153}{19200} \right) \\ &+ a_1 a_2 a_\tau \left(-\frac{9\zeta_3}{5} - \frac{1041}{320} \right) + a_2 a_b a_t \left(-\frac{9\zeta_3}{2} - \frac{2307}{32} \right) + a_1 a_b a_\tau \left(\frac{491}{120} - \frac{27\zeta_3}{5} \right) \\ &+ a_1 a_\tau^2 \left(-\frac{27\zeta_3}{5} - \frac{27}{16} \right) + a_1^2 a_\tau \left(-\frac{117n_G}{40} - \frac{807\zeta_3}{100} - \frac{4043}{640} \right) + a_2 a_\tau a_t \left(-9\zeta_3 - \frac{81}{4} \right) \\ &+ a_2^2 a_\tau \left(-\frac{21n_G}{8} - \frac{81\zeta_3}{4} + \frac{2121}{128} \right) + a_2^2 a_b \left(-\frac{69n_G}{16} - \frac{225\zeta_3}{8} + \frac{13653}{256} \right) \\ &+ a_b^2 a_t \left(\frac{825}{8} - 48\zeta_3 \right) + a_2^2 a_t \left(-\frac{351n_G}{16} - \frac{729\zeta_3}{8} + \frac{49239}{256} \right) - \frac{45a_\tau \hat{\lambda}^2}{2} - \frac{291a_b \hat{\lambda}^2}{4} \\ &+ 45a_2 \hat{\lambda}^2 + 9a_1 \hat{\lambda}^2 + 30a_\tau a_t \hat{\lambda} + 93a_b a_t \hat{\lambda} - \frac{135}{2} a_2 a_t \hat{\lambda} - \frac{127}{10} a_1 a_t \hat{\lambda} \\ &+ 15a_\tau^2 \hat{\lambda} + 15a_b^2 \hat{\lambda} - \frac{171a_2^2 \hat{\lambda}}{16} + \frac{117a_1 a_2 \hat{\lambda}}{40} - \frac{1089a_1^2 \hat{\lambda}}{400} + \frac{21a_\tau a_t^2}{2} + \frac{739a_b a_t^2}{16} - \frac{1593a_2 a_t^2}{16} \\ &- \frac{2437a_1 a_t^2}{80} + \frac{207a_\tau^2 a_t}{8} + \frac{7a_b a_\tau a_t}{2} + \frac{53a_b a_\tau^2}{4} + 22a_b^2 a_\tau\end{aligned}\quad (13)$$

with n_G corresponding to the number of SM generations and $\zeta_3 = \zeta(3)$. Having in mind the anomaly cancellation conditions, the result is only valid for the SU(3) color group, so we substitute all the color invariants by the corresponding values ($C_F = 4/3$, $N_c = 3$, $C_A = 3$).

It should be noted that the expressions are free from gauge-fixing parameters ξ_G, ξ_W and ξ_B which are present in the renormalization constants for the considered two- and three-point Green functions. The one- and two-loop corrections, given in Eqs. (11) and (12), are in full agreement with Refs. [27, 31, 32, 38]. The contribution (13) coincides with the result of Ref. [35] in the limit of vanishing couplings a_1, a_2, a_b , and a_τ (first two lines of Eq. (13)).

In order to estimate the numerical impact of the calculated corrections, we use known values of the parameters at the M_Z scale (e.g., from Ref. [38]) and compute the values of the considered three-loop contributions to the Yukawa coupling beta-functions. In what follows, the largest contributions that account for at least 99% of the total three-loop corrections to the beta-functions are shown:

$$\frac{\beta_t^{(3)}}{\beta_t^{(3)}(M_Z)} \simeq \boxed{1.59A_s^3 - 0.60A_s^2A_t + 0.17A_sA_t^2 - 0.07A_t^2\Lambda} - 0.05A_2A_t^2 - \boxed{0.04A_t^3}, \quad (14)$$

$$\frac{\beta_b^{(3)}}{\beta_b^{(3)}(M_Z)} \simeq 0.83A_s^3 + 0.17A_s^2A_t + 0.03A_2A_sA_t - 0.03A_2A_s^2 - 0.01A_t^3, \quad (15)$$

$$\begin{aligned} \frac{\beta_\tau^{(3)}}{\beta_\tau^{(3)}(M_Z)} &\simeq 1.25A_s^2A_t - 0.35A_sA_t^2 + 0.17A_t^3 + 0.07A_t^2\Lambda - 0.06A_t\Lambda^2 \\ &\quad - 0.05A_2A_t^2 + 0.04A_2^3 - 0.04A_2^2A_s - 0.02A_2^2A_t, \end{aligned} \quad (16)$$

where

$$A_i = \frac{a_i}{a_i(M_Z)}, \quad \Lambda = \frac{\lambda}{\lambda(M_Z)}. \quad (17)$$

The known results from Ref. [35] are highlighted with boxes in Eq. (14).

It is clear that all the corrections are dominated by the terms proportional to the top Yukawa and the strong coupling constants. However, one can see that the $a_s^2a_2$ contribution to the top Yukawa beta-function (14), which was not taken into account in Ref. [35], is comparable with the a_t^3 term.

To conclude, in this paper we present for the first time the expressions for three-loop Yukawa beta-functions. The latter can be used together with the results of Refs. [33, 34] and [35] in a precise RGE analysis not only of the SM itself but also of its extensions which introduce New Physics at a scale very much separated from the electroweak one. However, one should keep in mind that the full three-loop beta-function for the quartic coupling and the anomalous dimension of the Higgs mass parameter are still missing and require a dedicated study which will be presented elsewhere.

Acknowledgments

The authors would like to thank M. Kalmykov for stimulating discussions. This work is partially supported by RFBR grants 11-02-01177-a, 12-02-00412-a, RSGSS-4801.2012.2.

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